

for $y \geq y_c$, $\rho = \rho_e$ and $u = u_e$. It is assumed also that $\partial(\rho_e u_e)/\partial y = 0$ for $y \geq y_c$.

Continuity is applied to yield the mass balance within the control surface,

$$\int_0^{y_c} (\rho u)_2 dy = \int_0^{y_c} (\rho u)_1 dy + \int_{x_1}^{x_2} (\rho_w v_w - \rho_e v_e) dx \quad (1)$$

and the usual definition of displacement thickness δ^* is introduced through the equation

$$\begin{aligned} \int_0^{y_c} (\rho u) dy &= \rho_e u_e \left[y_c - \int_0^{y_c} \left(1 - \frac{\rho u}{\rho_e u_e} \right) dy \right] \\ &= \rho_e u_e [y_c - \delta^*] \end{aligned} \quad (2)$$

When Eq. (2) is substituted into Eq. (1) and the limit as $(x_2 - x_1) \rightarrow 0$ is taken, the result is

$$\rho_e v_e|_{y=y_c} = \rho_e u_e (d\delta^*/dx) + \rho_w v_w - (y_c - \delta^*) [d(\rho_e u_e)/dx] \quad (3)$$

The first two terms on the right-hand side of Eq. (3) represent the vertical component of mass flow induced by the boundary layer. The last term on the right side [i.e., $-(y_c - \delta^*)d(\rho_e u_e)/dx$] is the vertical component of mass flow which would exist at a normal distance $(y_c - \delta^*)$ from a solid body for the case of inviscid flow with a surface axial mass flow gradient $\partial(\rho_e u_e)/\partial x$ and $\partial(\rho_e u_e)/\partial y = 0$.[†] This can be seen from the continuity equation of the inviscid flow,

$$\partial(\rho_e v_e)/\partial y = -\partial(\rho_e u_e)/\partial x \quad (4)$$

which leads to

$$\rho_e v_e|_{y=y_c} = \rho_e v_e|_{y=\delta^*} - \int_{\delta^*}^{y_c} \frac{\partial}{\partial x} (\rho_e u_e) dy \quad (5)$$

Now for $\partial(\rho_e u_e)/\partial x$ independent of y , Eq. (5) becomes

$$\rho_e v_e|_{y=y_c} = \rho_e v_e|_{y=\delta^*} - (y_c - \delta^*) [d(\rho_e u_e)/dx] \quad (6)$$

Thus, this additional vertical-mass-flow term is associated with the inviscid flow and must be neglected when computing the boundary layer displacement effect. (Note also that this term varies linearly with y_c and therefore is not defined uniquely.)

Neglecting the last term on the right-hand side of Eq. (3) and dividing by $\rho_e u_e$ gives the incremental flow angle induced by the boundary layer:

$$\frac{v_e}{u_e} \cong \theta_w \cong \frac{d\delta^*}{dx} + \frac{\rho_w v_w}{\rho_e u_e} \quad (7)$$

The effective boundary layer displacement thickness is

$$\int_0^x \theta_w dx \cong \Delta^* = \delta^* + \int_0^x \frac{\rho_w v_w}{\rho_e u_e} dx \quad (8)$$

The point emphasized is that the term δ^* is calculated from the boundary layer density and velocity profiles under the influence of blowing as obtained by the appropriate boundary layer solution, but the displacement effect on the external flow should include the additional term involving injected mass flow. It is believed that the necessity for including this additional mass-flow term in the calculation of interaction effects on ablating bodies generally has not been recognized.

References

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[†] This interpretation of Eq. (3) was pointed out to the author by H. Mirels of the Laboratories Division, Aerospace Corporation.

Particle Size Analysis of Ammonium Perchlorate by Liquid Sedimentation

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A method is described for obtaining particle size distributions in ammonium perchlorate and other finely divided materials from 400- μ diam to 1 μ and below. The method offers advantages including rapid operation and moderate equipment costs. Validity and accuracy of the method are found to be satisfactory for use in studies of particle size effects in solid propellant combustion.

Introduction

AMMONIUM perchlorate currently is used by many manufacturers as an oxidizer in composite solid propellants. In most cases this crystalline material is used partly or entirely in a finely ground condition. It long has been recognized that the average size and size distribution of the perchlorate particles are factors that influence the combustion behavior of such propellants. An experimental program has been completed recently at Princeton wherein the effects of these factors on burning rates were determined for various propellant compositions.^{1,2} At the outset of this program, it was necessary to obtain an instrument for measuring particle size distribution.

Measurement of fine particles is a complex subject, and methods that presently are known are numerous and varied.³ However, for particles in the so-called subsieve range, that is, particles less than 50 μ in diameter, methods most often used for measuring size distribution are based on the movement of particles through a fluid. This process lends itself to rapid and repeated analysis more than any other.[§] Instruments vary widely in nature, but the basic principle of operation is the same. The velocity with which a particle moves through a fluid is dependent upon the force acting upon the particle, the fluid viscosity, and the particle size and shape (for low Reynolds numbers). The effect of particle shape generally is avoided by assigning to each particle a diameter equal to that of a sphere of equal density which moves through the fluid with the same velocity. For nearly spherical particles, this device is satisfactory. Many instruments allow particles to fall (or rise) under the influence of gravity. This is called the sedimentation method. Others increase the force field by centrifugal action. In a constant force field, particle velocity is very sensitive to size, particularly in the Stokes regime, where velocity is proportional to the square of the particle diameter. Thus, instruments based on this principle are capable of indicating particle diameter with great accuracy.

In measurement of ammonium perchlorate particles, a number of special problems are encountered. Finely ground

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§ Rapid methods based on other principles do exist for measuring a mean particle size.